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Continuous creation of a vortex in a Bose–Einstein condensate with hyperfine spin $F = 2$

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Abstract

It is shown that a vortex can be continuously created in a Bose–Einstein condensate with hyperfine spin $F = 2$ in a Ioffe–Pritchard trap by reversing the axial magnetic field adiabatically. It may be speculated that the condensate cannot be confined in the trap since the weak-field seeking state makes transitions to the neutral and the strong-field seeking states due to the degeneracy of these states along the vortex axis when the axial field vanishes. We have solved the Gross–Pitaevskii equation numerically with given external magnetic fields to show that this is not the case. It is shown that a considerable fraction of the condensate remains in the trap even when the axial field is reversed rather slowly. This scenario is also analysed in the presence of an optical plug along the vortex axis. Then the condensate remains within the $F_z = 2$ manifold, with respect to the local magnetic field, throughout the formation of a vortex and hence the loss of atoms does not take place.

1. Introduction

It has been observed that the Bose–Einstein condensate (BEC) of alkali atom gas becomes superfluid [1, 2]. Superfluidity of this system is different from the previously known superfluid ^{4}He in many aspects. For example, the BEC is a weak-coupling gas for which the Gross–Pitaevskii (GP) equation is applicable while superfluid ^{4}He is a strong-coupling system. One of the most remarkable differences is that the BEC has a spin degree of freedom originating from the hyperfine spin of the atom, and that this degree of freedom couples to external magnetic fields. Accordingly the order parameter of the condensate is also controlled

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at will by external magnetic fields. Superfluid ^{3}He also has similar internal degrees of freedom, which, however, are rather difficult to control by external fields [3].

Taking advantage of this observation, we proposed a simple method to create a vortex in a BEC with the hyperfine spin $F = 1$ [4, 5]; a vortex-free BEC is intertwined topologically by manipulating the magnetic fields in the Ioffe–Pritchard trap to form a vortex with the winding number 2. This is achieved by reversing the axial magnetic field adiabatically while the planar quadrupole field is kept fixed.

In the present paper, a similar scenario is analysed for a BEC with $F = 2$, taking ^{87}Rb as an example. The difference between the present case and that for $F = 1$ will be emphasized in our analysis. In the next section, we briefly review the order parameter of BEC with hyperfine spin $F = 2$ and the GP equation which describes the time evolution of the order parameter. In section 3, the ground state order parameter of the BEC in the weak-field seeking state (WFSS) confined in a harmonic potential is obtained. Then the time evolution of the condensate, as the axial field is adiabatically reversed is studied by solving the GP equation numerically. Cases with different reverse time are analysed to find the best possible reversing time for which the fraction of the remaining condensate in the vortex state is maximized. It is shown that the condensate at the end of this scenario has the winding number 4. In section 4, the GP equation is solved in the presence of an optical plug along the vortex axis. The BEC remains in the $F_z = 2$ WFSS, with respect to the local magnetic field, throughout the development, and hence no atoms will be lost during the formation of a vortex. Section 5 is devoted to conclusions and discussions.

2. Order parameter of F = 2 BEC

2.1. General $F = 2$ condensate and Gross–Pitaevskii equation

Suppose a uniform magnetic field $\boldsymbol{B}$ parallel to the z axis is applied to a BEC of alkali atoms with the hyperfine spin $F = 2$. Then the hyperfine spin state of the atom is quantized along this axis; the eigenvalue m of F_z takes a value $-2 \leqslant m \leqslant +2$, where $F_z|m\rangle = m|m\rangle$. Let us introduce the following conventions:

$$|2\rangle = \begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, \qquad |1\rangle = \begin{pmatrix}0\\1\\0\\0\\0\end{pmatrix}, \qquad |0\rangle = \begin{pmatrix}0\\0\\1\\0\\0\end{pmatrix},$$

$$|-1\rangle = \begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}, \qquad |-2\rangle = \begin{pmatrix}0\\0\\0\\0\\1\end{pmatrix}.$$

The order parameter $|\Psi\rangle$ is expanded in terms of $|m\rangle$ as

$$|\Psi\rangle = \sum_{m=-2}^{2} \Psi_m |m\rangle = (\Psi_2, \Psi_1, \Psi_0, \Psi_{-1}, \Psi_{-2})^{\mathrm{T}}, \tag{1}$$

where T denotes the transpose.

The representation of the angular momentum operators F_k $(k = x, y, z)$ for $F = 2$ is easily obtained from the well-known formulae

$$\langle 2, m|F_+|2, m'\rangle = \sqrt{(2-m)(3+m)}\,\delta_{m,m'+1},$$

$$\langle 2, m|F_-|2, m'\rangle = \sqrt{(2+m)(3-m)}\delta_{m,m'-1},$$

$$\langle 2, m|F_z|2, m'\rangle = m\delta_{m,m'},$$

where $F_\pm = F_x \pm \mathrm{i}F_y$.

The dynamics of the condensate in the limit of zero temperature is given, within the mean field approximation, by the time-dependent GP equation with spin degrees of freedom. This equation, obtained by Ciobanu *et al* [6] (see also [7]), is written in components Ψ_m as

$$\begin{aligned}\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi_m = &\left[-\frac{\hbar^2}{2M}\nabla^2 + V(r)\right]\Psi_m + g_1|\Psi_n|^2\Psi_m + g_2[\Psi_n^\dagger(F_k)_{np}\Psi_p](F_k)_{mq}\Psi_q \\ &+ 5g_3\Psi_n^\dagger\langle 2m2n|00\rangle\langle 00|2p2q\rangle\Psi_p\Psi_q + \tfrac{1}{2}\hbar\omega_{Lk}(F_k)_{mn}\Psi_n,\end{aligned} \tag{2}$$

where summations over $k = x, y, z$ and $-2 \leqslant n, p, q \leqslant 2$ are understood. Here, M is the mass of the atom and $V(r)$ is the possible external potential. The Larmor frequency is defined as $\hbar\omega_{Lk} = \gamma_\mu B_k$, where $\gamma_\mu \simeq \mu_B$ is the gyromagnetic ratio of the atom and μ_B is the Bohr magneton. The interaction parameters are expressed in terms of the s-wave scattering length a_F, F being the total hyperfine spin of the two-body scattering state, and are given by [6]

$$g_1 = \frac{4\pi\hbar^2}{M}\frac{4a_2+3a_4}{7} \qquad g_2 = \frac{4\pi\hbar^2}{M}\frac{a_2-a_4}{7} \qquad g_3 = \frac{4\pi\hbar^2}{M}\left(\frac{a_0-a_4}{5} - \frac{2a_2-2a_4}{7}\right), \tag{3}$$

where $a_0 = 4.73$ nm, $a_2 = 5.00$ nm and $a_4 = 5.61$ nm for ^{87}Rb atoms. This should be compared with $F = 1$ BEC where there are only two types of scattering state and hence two interaction terms in the GP equation.

2.2. Weak-field seeking state

Suppose a strong magnetic field $\boldsymbol{B}$ is applied along the z axis. Then the components with $F_z = 1$ and 2 are in the WFSS while those with $F_z = -1$ and -2 are in the strong-field seeking state (SFSS). The presence of two WFSSs leads to an interesting two-component vortex that is not observed in $F = 1$ BEC, as we see in the next section. The energy of the state with $F_z = 0$ is independent of the magnetic field and will be called the neutral state (NS) hereafter. Suppose a uniform condensate is in the state with $F_z = 2$. The order parameter of the condensate takes the form

$$|\Psi_0\rangle = f_0(1, 0, 0, 0, 0)^{\mathrm{T}} \tag{4}$$

where $|f_0|^2$ is the number density of the condensate. Now let us consider a state which is quantized along an arbitrary local magnetic field:

$$\boldsymbol{B}(\boldsymbol{r}) = B\begin{pmatrix}\sin\beta\cos\alpha \\ \sin\beta\sin\alpha \\ \cos\beta\end{pmatrix}. \tag{5}$$

Let $F_B \equiv \boldsymbol{B}\cdot\boldsymbol{F}/B$ be the projection of the hyperfine spin vector along the local magnetic field. The WFSS $|\Psi\rangle$ which satisfies $F_B|\Psi\rangle = +2|\Psi\rangle$ is obtained by rotating $|\Psi_0\rangle$ by Euler angles α, β and γ and is given by

$$\begin{aligned}|\Psi(\boldsymbol{r})\rangle &= \exp(-\mathrm{i}\alpha F_z)\exp(-\beta F_y)\exp(-\mathrm{i}\gamma F_z)|\Psi_0\rangle \\ &= f_0\mathrm{e}^{-2\mathrm{i}\gamma}\begin{pmatrix}\mathrm{e}^{-2\mathrm{i}\alpha}\cos^4\frac{\beta}{2} \\ 2\mathrm{e}^{-\mathrm{i}\alpha}\cos^3\frac{\beta}{2}\sin\frac{\beta}{2} \\ \sqrt{6}\cos^2\frac{\beta}{2}\sin^2\frac{\beta}{2} \\ 2\mathrm{e}^{\mathrm{i}\alpha}\cos\frac{\beta}{2}\sin^3\frac{\beta}{2} \\ \mathrm{e}^{2\mathrm{i}\alpha}\sin^4\frac{\beta}{2}\end{pmatrix} \equiv f_0|v\rangle.\end{aligned} \tag{6}$$

The GP equation restricted within the WFSS is obtained by substituting equation (6) into equation (2), see the next section.

3. Vortex formation without optical plug

The formation of a vortex in the $F = 2$ condensate is analysed in this and the next sections. In the present section, we study the scenario without an optical plug along the vortex axis. Although some fraction of the condensate is lost from the trap in this scenario, the experimental set-up will be much easier without introducing an optical plug. In fact, it will be shown below that a considerable amount of condensate remains in the trap by properly choosing the time dependence of the magnetic field.

3.1. Magnetic fields

Suppose a condensate is confined in a Ioffe–Pritchard trap. It is assumed that the trap is translationally invariant along the z direction and rotationally invariant around the z axis. The quadrupole magnetic field of the trap takes the form

$$\boldsymbol{B}_\perp(\boldsymbol{r}) = B_\perp(r)\begin{pmatrix}\cos(-\phi)\\ \sin(-\phi)\\ 0\end{pmatrix}, \tag{7}$$

where ϕ is the polar angle. The magnitude $B_\perp(r)$ is proportional to the radial distance r near the origin; $B_\perp(r) \sim B'_\perp r$. The uniform time-dependent field

$$\boldsymbol{B}_z(t) = \begin{pmatrix}0\\ 0\\ B_z(t)\end{pmatrix} \tag{8}$$

is also applied along the z axis to prevent Majorana flips from taking place at $r \sim 0$ where $\boldsymbol{B}_\perp$ vanishes. Now the total magnetic field is given by

$$\boldsymbol{B}(\boldsymbol{r}, t) = \boldsymbol{B}_\perp(\boldsymbol{r}) + \boldsymbol{B}_z(t) = \begin{pmatrix}B_\perp(r)\cos(-\phi)\\ B_\perp(r)\sin(-\phi)\\ B_z(t)\end{pmatrix}. \tag{9}$$

Comparing this equation with equation (5), it is found that

$$\alpha = -\phi \qquad \beta = \tan^{-1}\left[\frac{B_\perp(r)}{B_z(t)}\right]. \tag{10}$$

It has been shown in the previous work for $F = 1$ BEC that a vortex-free condensate in the beginning will end up with a condensate with a vortex of winding number 2 if $\boldsymbol{B}_z$ reverses its direction while $\boldsymbol{B}_\perp$ is kept unchanged [4, 5]. We expect the same magnetic field manipulation to lead to the vortex formation in a BEC with $F = 2$. The uniform axial field $\boldsymbol{B}_z(t)$ must reverse its direction as

$$B_z(t) = \begin{cases} B_z(0)\left(1 - \dfrac{2t}{T}\right) & 0 \leqslant t \leqslant T \\ -B_z(0) & T < t \end{cases} \tag{11}$$

to create a vortex along the z axis. This gives a 'twist' to the condensate, leading to the formation of a vortex with winding number 4, see below.

Before we start the detailed analysis, it will be useful to outline the idea underlying our scenario. Suppose one has WFSS with $F_B = 2$ in the trap at $t = 0$. The magnetic field at $r \sim 0$ points in the $+z$ direction (i.e. $\beta \sim 0$) and hence the WFSS takes the form Ψ_0 of equation (4). Then the angle γ must satisfy $\gamma = -\alpha$ for the BEC to be vortex-free, see equation (6). The field $B_z(t)$ vanishes at $t = T/2$, for which $\beta = \pi/2$, and the hyperfine spin is parallel to the quadrupole field $\boldsymbol{B}_\perp$. Accordingly one must choose $\alpha = -\phi$ for this condition to be satisfied, see equations (9) and (10). This also implies $\gamma = +\phi$. When the field B_z is completely reversed at $t = T$, the magnetic field at $r \sim 0$ points down and hence $\beta \sim \pi$ there. Substituting $\alpha = -\gamma = \phi$ and $\beta = \pi$ into equation (6), one finds the order parameter at $t = T$:

$$|\Psi\rangle = f_0 \mathrm{e}^{-2\mathrm{i}\phi}(0, 0, 0, 0, \mathrm{e}^{-2\mathrm{i}\phi})^{\mathrm{T}}, \tag{12}$$

which shows that a vortex with winding number 4 has been created.

3.2. *Initial state*

Suppose a vortex-free BEC is confined in a Ioff–Pritchard trap, whose magnetic field takes the form (9) and that the condensate is in the eigenstate $F_B = 2$ with respect to the local magnetic field $\boldsymbol{B}$ with $B_z = B_z(t = 0)$. The condensate wavefunction is then obtained by solving the stationary state GP equation. Substitution of equation (6) with $\alpha = -\gamma = -\phi$ into equation (2) yields

$$-\frac{\hbar^2}{2M}\nabla^2(f_0 v_m) + (g_1 + 4g_2) f_0^3 v_m + \hbar\omega_L f_0 v_m = \mu f_0 v_m,$$

where we have put $\Psi_m \equiv f_0 v_m$. Note that the g_3 term vanishes identically for the present state. The condensate wave amplitude $f_0(r)$ is taken to be a real function without loss of generality. The eigenvalue μ is identified with the chemical potential. If one multiplies the above equation by $\{v_m\}^\dagger$ from the left and uses the identity $\sum_m |v_m|^2 = 1$ and other identities derived from this, one obtains the reduced GP equation for $f_0(r)$:

$$-\frac{\hbar^2}{2M}\left[f_0'' + \frac{f_0'}{r} + (v_m^* \nabla^2 v_m) f_0\right] + (g_1 + 4g_2) f_0^3 + \hbar\omega_L f_0 = \mu f_0, \tag{13}$$

where

$$v_m^* \nabla^2 v_m = \left[\frac{2}{r^2}(3\cos^2\beta - 5)\sin^2\frac{\beta}{2} - \beta'^2\right] \tag{14}$$

comes from the rotation of the five-dimensional local orthonormal frame that defines the order parameter. The reduced GP equation looks similar to the ordinary scalar GP equation except that there is an extra term β'^2 in $v_m^* \nabla^2 v_m$.

It is convenient to introduce the energy scale $\hbar\omega$ and the length scale a_{HO} defined by

$$\omega = \sqrt{\frac{\gamma_\mu}{M B_z(0)}} B'_\perp \qquad a_{\mathrm{HO}} = \sqrt{\frac{\hbar}{M\omega}}. \tag{15}$$

For typical values $B_z(0) = 1$ G, $B'_\perp = 200$ G cm^{-1} for ^{87}Rb, one obtains $\hbar\omega \simeq 1.69 \times 10^{-24}$ erg and $a_{\mathrm{HO}} \simeq 0.68\ \mu$m. After scaling all the physical quantities by these units, one obtains the dimensionless form of the reduced GP equation:

$$-\frac{1}{2}\left[\tilde{f}_0'' + \frac{\tilde{f}_0'}{\tilde{r}} + \left[\frac{2}{\tilde{r}^2}(3\cos^2\tilde{\beta} - 5)\sin^2\frac{\tilde{\beta}}{2} - \tilde{\beta}'^2\right]\tilde{f}_0\right] + (\tilde{g}_1 + 4\tilde{g}_2)\tilde{f}_0^3 + \tilde{\omega}_L \tilde{f}_0 = \tilde{\mu}\tilde{f}_0, \tag{16}$$

where the dimensionless quantities are denoted by a tilde. For example, $\tilde{r} = r/a_{\mathrm{HO}}$, $\tilde{f}_0 = f_0 a_{\mathrm{HO}}^{3/2}$ and $\tilde{g}_k = g_k/(\hbar\omega a_{\mathrm{HO}}^3)$. The tilde will be dropped hereafter unless otherwise stated explicitly.

Figure 1 shows the ground state condensate wavefunction obtained by solving equation (16) numerically. We have chosen $f_0(r = 0) = 6$ which yields the central

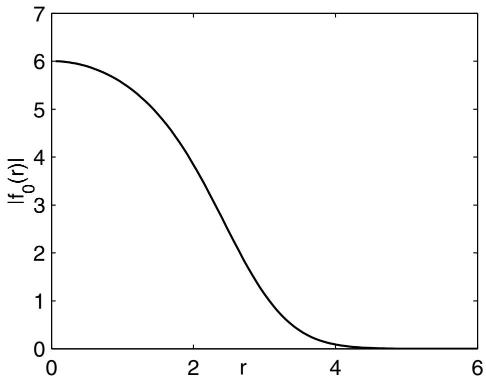

Figure 1. The initial condensate wavefunction $f_0(r)$ in dimensionless form. The radial coordinate r is also dimensionless.

density $n_0 \sim 1.17 \times 10^{14}$ cm^{-3}. This is roughly of the same order as that realized experimentally. The difference between the chemical potential and the Larmor energy at the origin is $\delta\mu = \mu - \omega_L = 3.95$, which amounts to $\delta\mu = 6.68 \times 10^{-24}$ erg in dimensional form.

3.3. Time development

Now the time-dependent GP equation (2) is solved numerically with the initial condition $\Psi_m = f_0(r)v_m$, with f_0 having been obtained in the previous subsection. We have introduced a tanh-shaped cutoff to mimic the loss of atoms from the trap; particles reaching $L \gg a_{\mathrm{HO}}$ vanish from the system. We have made several choices of the reversing time T and maximized the fraction of the condensate left in the trap in the final equilibrium state. The details of the algorithm are given in [5] and will not be repeated here.

Figure 2 shows the wavefunctions $|\Psi_m|$ for the choice $T/\tau = 1000$, where $\tau = 2\pi/\omega_L$ is the timescale set by the Larmor frequency at $t = 0$ and $r = 0$. One obtains $\tau \sim 7.14 \times 10^{-7}$ s for the parameters given in the previous subsection. The parameter τ is expected to be the measure of the adiabaticity. There are two WFSSs possible for $F = 2$, those with $F_B = 2$ and 1. It turns out that the final vortex state is a mixture of these two states. When the axial field $B_z(t)$ vanishes at $t = T/2$, the gaps among WFSSs, SFSSs and NS disappear at $r = 0$ and the level crossing takes place there. Then the adiabatic assumption breaks down and some fraction of the condensate transforms into SFSSs and NS as well as $F_B = 1$ WFSS. Those components in SFSSs and NS eventually leave the trap and the final condensate is made of $F_B = 2$ and 1 components. It is a remarkable feature of the $F = 2$ BEC, compared to its $F = 1$ counterpart, that the vortex state thus created is a mixture of these two WFSSs. The composite nature of the final vortex state is best revealed by projecting $|\Psi(\boldsymbol{r})\rangle$ to $F_B = 1$ and 2 states. Let $|v\rangle$ be the vector defined in equation (6) and $|u\rangle = \exp(-\mathrm{i}\alpha F_z)\exp(-\beta F_y)\exp(-\mathrm{i}\gamma F_z)|1\rangle$. Then $\Pi_2(\boldsymbol{r}) \equiv \langle v(\boldsymbol{r})|\Psi(\boldsymbol{r})\rangle$ and $\Pi_1(\boldsymbol{r}) \equiv \langle u(\boldsymbol{r})|\Psi(\boldsymbol{r})\rangle$ depict the projected amplitudes of $|\Psi(\boldsymbol{r})\rangle$ to the local $F_B = 2$ and 1 state, respectively. These amplitudes are shown in figure 3 for $|\Psi(\boldsymbol{r})\rangle$ at $t = 10$ T. It is interesting to note that the $F_B = 2$ component has a winding number 4 while $F_B = 1$ has 3.

The fraction of the condensate left in the trap at time t has been plotted in figure 4 for $T/\tau = 1000$. It should be noted that $\sim 2/5$ of the condensate is left in the trap when the system reaches an equilibrium at $t \gg T$.

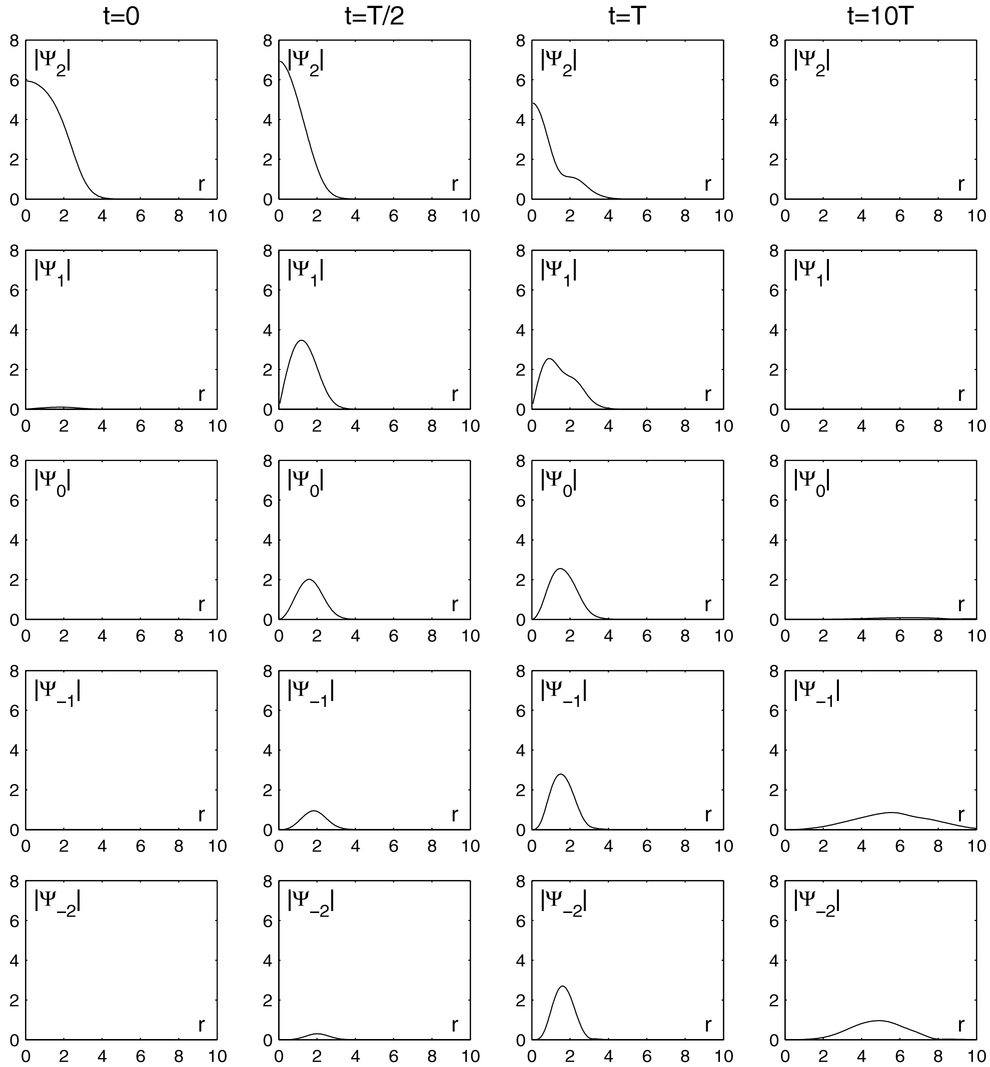

Figure 2. Time dependence of the order parameter $|\Psi_m|$ for the reversing time $T/\tau = 1000$.

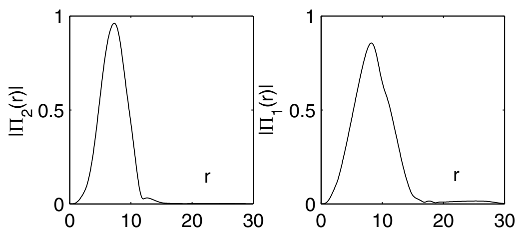

Figure 3. The projected amplitudes $|\Pi_2(r)|$ and $|\Pi_1(r)|$ obtained from the order parameter $|\Psi(r)\rangle$ at $t = 10$ T.

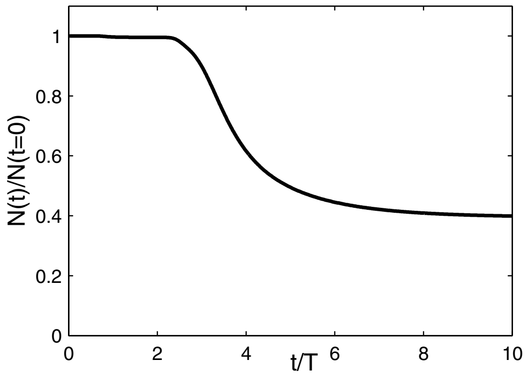

Figure 4. The fraction of the condensate left in the trap, as a function of the dimensionless time t/τ, for the reversing time $T/\tau = 1000$.

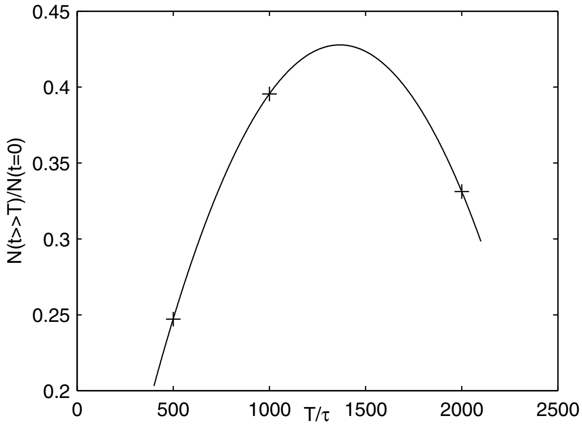

Figure 5. The fraction of the condensate left in the trap, as a function of T/τ, when the BEC reaches equilibrium at $t \gg T$. The curve is shown for a guide.

Figure 5 shows the fraction of the condensate left in the trap in the equilibrium state at $t \gg T$ for various T. It can be seen from this figure that a considerable amount of the condensate is left in the trap for a wide variety of reversing times T.

In the next section, we analyse the creation of a vortex in the presence of an optical plug along the centre of the system. It will be shown that the vortex thus created is made purely of $F_B = 2$ WFSS.

4. Vortex formation with optical plug

The loss of the condensate in the previous section takes place since the energy gaps among WFSSs, NS and SFSSs disappear at $r = 0$ when B_z vanishes at $t = T/2$. One may introduce an optical plug along the vortex axis to prevent the condensate from entering this 'dangerous' region. An optical plug may be simulated by a repulsive potential

$$V(r) = V_0 \exp\left(-\frac{r^2}{r_0^2}\right) \tag{17}$$

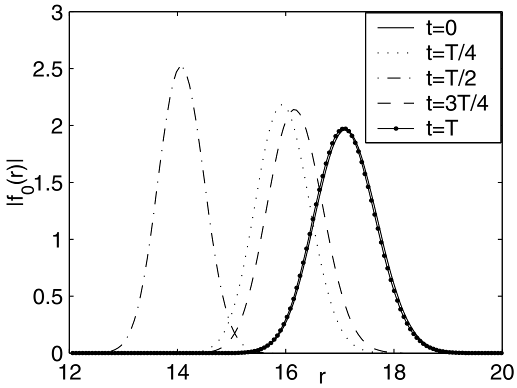

Figure 6. Time dependence of the condensate amplitude f_0 in the presence of the optical plug. The reversing time is $T/\tau = 10\,000$. The condensate amplitudes at $t = 0$ and T are almost degenerate.

where V_0 is determined by the power of the blue-detuned laser while r_0 is determined by its waist size. We take $V_0 = 9.27 \times 10^{-21}$ erg and $r_0 = 5$ μm in our computation below.

Now the time-independent GP equation is given by

$$-\frac{1}{2}\left[\tilde{f}_0'' + \frac{\tilde{f}_0'}{\tilde{r}} + \left[\frac{2}{\tilde{r}^2}(3\cos^2\tilde{\beta} - 5)\sin^2\frac{\tilde{\beta}}{2} - \tilde{\beta}'^2 + \tilde{V}(r)\right]\tilde{f}_0\right] + (\tilde{g}_1 + 4\tilde{g}_2)\tilde{f}_0^3 + \tilde{\omega}_L\tilde{f}_0 = \tilde{\mu}\tilde{f}_0 \tag{18}$$

in dimensionless form, where $\tilde{V}(r) = V(r)/\hbar\omega$. The angle β is given by $\beta(r) = \tan^{-1}[B_\perp(r)/B_z(0)]$. We will drop the tilde from dimensionless quantities hereafter unless otherwise stated. The ground state condensate wavefunction is obtained by solving this equation numerically. We find the relative chemical potential $\delta\mu = \mu - \omega_L = 173$, which amounts to $\delta\mu = 2.92 \times 10^{-22}$ erg in dimensional form, and the condensate wavefunction f_0 shown in figure 6.

The time-dependent GP equation

$$\mathrm{i}\frac{\partial f_0}{\partial t} = -\frac{1}{2}\left[f_0'' + \frac{f_0'}{r} + \left[\frac{2}{r^2}(3\cos^2\beta - 5)\sin^2\frac{\beta}{2} - \beta'^2 + V(r)\right]f_0\right] + (g_1 + 4g_2)f_0^3 + \omega_L f_0 \tag{19}$$

is solved with the initial wavefunction obtained above. Here $\beta = \beta(r,t) \equiv \tan^{-1}[B_\perp(r)/B_z(t)]$. Figure 6 shows the time dependence of the condensate amplitude as a function of time for $T/\tau = 10000$, namely $T = 71.14$ ms in dimensional form. Figure 7 shows the time dependence of the particle numbers of the components Ψ_m for the same choice of T. In contrast with the case without optical plug, the time dependence of the order parameter is independent of the choice of T, up to a global phase, so long as T/τ is large enough so that adiabaticity is observed.

The vortex thus obtained has a region near the origin ($r \sim 0$) where the condensate cannot approach due to the presence of the optical plug. The vortex current flows around a multiply connected region. This situation is analogous to the superconducting current flowing around a ring. It is natural to expect that a vortex without the optical plug may be obtained if one withdraws the optical plug after the persistent current is established at $t = T$. (Note that the optical plug has been introduced to prevent Majorana flips at $r \sim 0$ at $t \sim T/2$. Accordingly

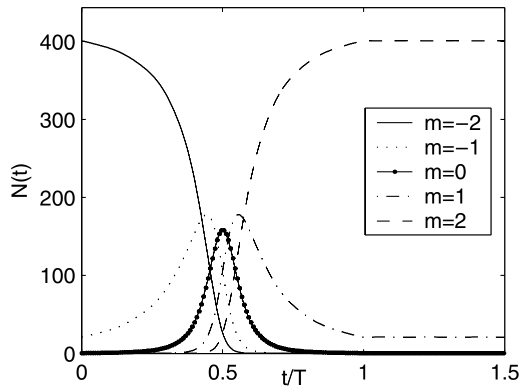

Figure 7. The time dependence of the particle numbers in unit length $N_m(t) = 2\pi \int |\Psi_m(r,t)|^2 r\,\mathrm{d}r$ for $T/\tau = 10\,000$.

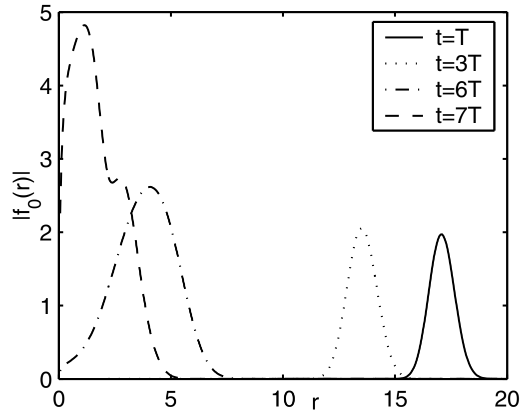

Figure 8. Time dependence of the condensate amplitude f_0 for the reversing time $T/\tau = 10\,000$. The potential decays exponentially with the time constant $t_0 = T$ for $t > T$, see equation (20).

the optical plug is not required any longer for $t \geqslant T$.) Let us suppose that the optical plug is slowly turned off after $t = T$ with the time constant t_0:

$$V(r,t) = \begin{cases} V_0 \exp(-r^2/r_0^2) & 0 < t < T \\ V_0 \exp(-r^2/r_0^2) \exp[-(t-T)/t_0] & T < t. \end{cases} \tag{20}$$

It is found that the condensate oscillates back and forth for small t_0. For sufficiently large t_0, however, the condensate smoothly rearranges itself to a vortex state without the optical plug. Figure 8 shows our numerical result for $T/\tau = t_0/\tau = 10\,000$, for which one still observes such oscillations.

A vortex with a winding number 4 is thus created without losing any atoms from the trap. It should be noted, however, that it is technically difficult, albeit not impossible [8], to introduce an optical plug with a few microns of radius along the centre of the BEC whose radial dimension without an optical plug is of the order of a few microns.

5. Conclusions and discussions

The formation of a vortex in a BEC with $F = 2$ in a Ioffe–Pritchard trap has been considered by fully utilizing the spinor degrees of freedom. It was shown that a vortex with winding number 4 is created continuously from a condensate without a vortex, by simply reversing the axial magnetic field $B_z(t)$. This scenario has been studied with and without an optical plug at the centre of the vortex. Some amount of the BEC is lost from the trap in the absence of an optical plug while no atoms are lost in the presence of it. Our numerical analysis shows that there remains a considerable fraction of BEC even without the optical plug. The introduction of an optical plug in a trapped BEC is difficult, albeit not impossible.

Our vortex has a large winding number 4 and is expected to be unstable against decay into four singly quantized vortices in the absence of an optical plug. Whether a vortex with such a large winding number may be observable depends on how large the lifetime of the metastable state is compared to the trapping time of the BEC. Our preliminary analysis of the Bogoliubov equation suggests that the lifetime is of the order of 100 ms and these highly quantized vortices exist for a considerable duration of time.

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Note added. After we submitted our manuscript, the MIT group reported the formation of vortices according to the present scenario [9]. They employed hyperfine spin states $F = 1$ and 2 of ^{23}Na and found that the vortex thus created had the winding number 2 in the former case while it was 4 in the latter case, consistent with our prediction. The vortex state has a considerably long lifetime, at least 30 ms after its formation, in spite of the higher winding number, which suggests that these vortices are rather stable. The stability analysis of highly quantized vortices is outside the scope of the present work and will be published elsewhere.

References

[1] Inguscio M, Stringari S and Wieman C E (ed) 1999 *Bose–Einstein Condensation in Atomic Gases* (Amsterdam: IOS Press)
[2] Martellucci S, Chester A N, Aspect A and Inguscio M (ed) 2000 *Bose–Einstein Condensates and Atomic Lasers* (New York: Kluwer)
[3] Vollhardt D and Wölfle P 1990 *The Superfluid Phases of Helium* vol 3 (London: Taylor and Francis)
[4] Isoshima T, Nakahara M, Ohmi T and Machida K 2000 *Phys. Rev.* A **61** 10000
[5] Ogawa S-I, Möttönen M, Nakahara M, Ohmi T and Shimada H 2002 *Phys. Rev.* A **66** 013617
[6] Ciobanu C V, Yip S-K and Ho T-L 2000 *Phys. Rev.* A **61** 033607
[7] Koashi M and Ueda M 2000 *Phys. Rev. Lett.* **84** 1066
[8] Davis K B *et al* 1995 *Phys. Rev. Lett.* **75** 3969
[9] Leanhardt A E *et al* 2002 *Phys. Rev. Lett.* **89** 190403